Synchronization in systems with linear, yet nonreciprocal interactions

C. Räth[1], M. Haslauer[2], I. Laut[1]
[1]German Aerospace Center (DLR)
[2]Ludwig-Maximilians-Universität München

Synchronization of oscillatory subsystems is a widespread phenomenon with examples in biology, neuroscience, chemistry and physics. A common feature of all these systems is that they are somehow connected to the synchronization theory of nonlinear limit cycle oscillators. It is thus often argued that the presence of nonlinearities is a necessary prerequisite for synchronization to happen. In this contribution we study synchronization in complex plasmas, which are plasmas containing microparticles in addition to ions, electrons and neutral gas atoms of the plasma. These microparticles show a strong (nonlinear) Coulomb interaction. The systems can form so-called plasma crystals. In experiments under gravity conditions the plasma crystals are two-dimensional hexagonal crystalline structures. They show a plasma specific melting mechanism called mode-coupling instability (MCI). MCI is a consequence of the effective nonreciprocal interactions of the microparticles. Nonreciprocal means that Newton's third law "actio = reactio" is violated if focusing on the microparticles. This is possible because the effective interactions of the microparticles are mediated by a nonequilibrium environment of flowing plasma ions. Recently, the observation of synchronized particle motion during such a mode-coupling instability induced melting of a two-dimensional plasma crystal was reported. In order to disentangle the effects of nonlinearity and nonreciprocity on the emergence of synchronization, we solved numerically the nonlinear and the linearized system for identical lattice and system configurations, where the interaction force was linearized around the equilibrium configuration of the crystal. Analyzing the onset of the synchronization with a newly developed, Kuramoto-inspired order parameter reveals that a linearized version of the interaction model exhibits exactly the same synchronization patterns as the complete nonlinear interaction model. Further theoretical considerations then naturally show that the nonreciprocal interactions of the microparticles provide a mechanism for the selection of dominant wave modes causing the system to show synchronized motion. In conclusion, we demonstrate numerically and analytically that - in contrast to common belief - also linear systems can synchronize and that the nonreciprocity of the interaction is the decisive property for a linear n-body system to synchronize.

[l] L. Couedel et al., Phys Rev. E **89**, 053108 (2014).
[2] I. Laut et al., Europhys. Lett. **110**, 65001 (2015).
[3] I. Laut et al., Phys Rev. E **93**, 013204 (2016).